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Effects of non-factorizable metric on neutrino oscillation inside supernova

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Abstract

In this paper we construct the interaction potential between the dense supernova core and neutrinos due to the exchange of light radions in the Randall-Sundrum scenario. We then show that the radion exchange potential affects the neutrino oscillation phenomenology inside the supernova significantly if the radion mass is less than around 1 GeV. In order that the Bethe-Wilson mechanism for heating the envelope and r-process neucleosynthesis be operative, the radion mass must be greater than 1 GeV. Bounds on the radion mass of the same order of magnitude can also be derived from TASSO and CLEO data on B decays.

I. Introduction

Several radical proposals based on higher dimensional spacetime have been put forward recently to explain the large hierarchy between the Planck scale and the electroweak (EW) scale. Among them the Randall-Sundrum (RS) scenario [1] is particularly attractive since it proposes a higher dimensional spacetime with a non-factorizable five dimensional metric of the form

$$ds^2 = e^{-2kr_c|\theta|}\eta_{\mu\nu}dx^\mu dx^\nu + r_c^2 d\theta^2 \tag{1}$$

where k is a scale of the order of the Planck mass, x^μ are the Lorentz coordinates of four dimensional surfaces of constant θ. $-\pi \leq \theta \leq \pi$ is the angular coordinate of the extra dimension which is an $\frac{S^1}{Z_2}$ space with orbifold symmetry. The points (x, θ) and (x, -θ) are therefore identified. The compactification radius r_c is the vacuum expectation value (vev) of the modulus field $T(x)$. Two 3-branes, the hidden and visible branes extending in the x^μ directions are located at the orbifold fixed points 0 and π. The existence of the modulus field is therefore the most direct and straightforward result of non-factorizable metric. In other words the effects of the nonfactorizable metric on the low energy phenomenology of the visible brane can be described in terms of the couplings of the modulus field to the SM fields. Randall and Sundrum showed [1] that the particular nonfactorizable metric of eqn (1) satisfies the five dimensional Einstein equations for this set up if the vacuum energies of the two 3-branes and the bulk cosmological constant are related in a particular way through the single scale factor k.

In the original proposal of Randall and Sundrum the modulus field was massless and had zero potential. The vev of the modulus was therefore not stabilized. Clearly a massless radion with TeV scale couplings is phenomenologically unacceptable since it would lead to a long range universal force that is 32 orders of magnitude stronger than gravity. Goldberger and Wise [2] showed that the modulus of RS scenario can be stabilized by introducing a scalar field in the bulk with suitable interaction potentials on the two 3-branes. In this work we shall assume that the modulus is stabilized by Goldberger-Wise mechanism and has a non-zero mass. By considering fluctuations of the radion field ϕ (which is related to the modulus field T(x) through the eqn $\phi(x) = fe^{-k\pi T(x)}$ with $f \approx M_{pl}$) about its vev it was shown in [3] that the radion couples to the visible brane matter through the trace

of the energy momentum tensor

$$L_I = T^\mu_\mu \frac{\tilde{\phi}}{\langle\phi\rangle} \tag{2}$$

Here T^μ_μ is the trace of the energy momentum tensor of the of visible brane matter. $\tilde{\phi} = \phi - \langle\phi\rangle$ is the fluctuation of the radion field from its vev $\langle\phi\rangle$. In the RS scenario $\langle\phi\rangle$ is in the TeV range for $kr_c \approx 12$ which is needed in order to generate the weak scale from the Planck scale through the exponential warp factor. Note that the radion coupling to matter fields on the visible brane given by eqn (2) is universal since it arises from the non-factorizable metric. The radion therefore always couples to the trace of the energy momentum tensor of the relevant fields or effective degrees of freedom on the visible brane. Inside the supernova the relevant matter fields are the nucleons and the radion coupling to them is given by

$$L_{\phi N} = \frac{\langle (T^\alpha_\alpha)_m \rangle}{\langle\phi\rangle}\tilde{\phi} \approx \frac{\rho_m}{\langle\phi\rangle}\tilde{\phi} \tag{3}$$

Here we have treated the nucleons inside the supernova as non relativistic. ρ_m is the matter density for non relativistic supernova matter. On the contrary the higgs boson has tree level couplings only to SM fermions and weak gauge bosons. The higgs coupling to nucleons arises from two gluon intermediate state which are emitted from the vertices of a triangular top quark loop. The higgs boson is connected to the third vertex. Effectively one integrates over virtual heavy quark fluctuations (which occur at distance scales of $\frac{1}{m_Q} \ll 1$ fm) to get the higgs coupling to nucleons. The usual loop suppression factor makes the higgs coupling to nucleons much smaller than that of the radion coupling ($\frac{m_N}{\langle\phi\rangle}$). We shall therefore ignore the effect of higgs exchange between the supernova matter and neutrinos in comparision to the radion exchange potential. The radion coupling to SM fermions on the visible brane is given by

$$L_{\phi f} = \frac{m_f}{\langle\phi\rangle}\bar{f}f\tilde{\phi} \tag{4}$$

Neutrinos can however have both Dirac and Majorana mass. But the neutrino oscillation probability at high energies is the same for either type of neutrino mass. In this paper we shall assume for definiteness that the radion couples to neutrinos through their Dirac mass term.

II. The radion exchange potential for neutrinos inside supernova

In this section we shall construct the radion exchange potential for neutrinos inside the supernova. Consider the scattering amplitude due to radion exchange between nonrelativistic supernova matter and neutrinos

$$M = \frac{1}{\langle\phi\rangle^2}\langle(T^\alpha_\alpha)_m\rangle\frac{1}{k^2 - m_\phi^2}\langle(T^\beta_\beta)_\nu\rangle \tag{5}$$

where m_ϕ is the radion mass. The trace of the stress tensor for non relativistic supernova matter is given by $\langle(T^\alpha_\alpha)_{mat})\rangle = \rho_m$. The trace of the energy momentum tensor for neutrinos is given by $\langle(T^\beta_\beta)_\nu\rangle = m_\nu\bar{\nu}(p)\nu(p)$. Here we shall assume forward scattering of neutrinos i.e $k \ll p$.

The exchange of radions between the supernova matter and neutrinos therefore generates an interaction potential for neutrinos which is given by

$$V_\nu = -\frac{1}{\langle\phi\rangle^2 m_\phi^2}m_\nu\rho_m \tag{6}$$

Unlike the weak interaction potential the radion exchange potential (6) has the same sign for neutrinos and antineutrinos.

III. Effect of radion exchange potential on ν oscillation in supernova

Neutrino oscillations between two different mass eigenstates ν_1 and ν_2 is given by the equation

$$i\frac{d}{dt}\begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix} = \left(\begin{pmatrix}0 & 0\\ 0 & \frac{\Delta m^2}{2E} - \frac{\Delta m^2}{2m_\nu}\frac{\rho_m}{m_\phi^2\,\langle\phi\rangle^2}\end{pmatrix} + H_{weak}\right)\begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix} \tag{7}$$

where $\Delta m^2 = (m_2^2 - m_1^2)$. The weak interaction potential in the flavour basis (say ν_e, ν_μ) is

$$H_{weak}\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix} = \begin{pmatrix}\sqrt{2}G_F N_e & 0\\ 0 & 0\end{pmatrix}\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix} \tag{8}$$

We can write (7-8) in the flavour basis as

$$i\frac{d}{dt}\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix} = \begin{pmatrix}0 & b\\ b & a\end{pmatrix}\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix} \tag{9}$$

where

$$a = \left(\frac{\Delta m^2}{2E} - \frac{\Delta m^2}{2m_\nu} \frac{\rho_m}{2m_\phi^2 \langle \phi \rangle^2} \right) cos2\theta - \sqrt{2} G_F N_e \tag{10}$$

and

$$b = \left(\frac{\Delta m^2}{2E} - \frac{\Delta m^2}{2m_\nu} \frac{\rho_m}{2m_\phi^2 \ \langle \phi \rangle^2} \right) \ \frac{1}{2} sin2\theta \tag{11}$$

where θ is the vaccum mixing angle between ν_e and ν_μ and N_e is the number density of electrons. One can see from (10, 11) that the radion exchange potential will be significant in neutrino oscillation if

$$\frac{\Delta m^2}{2E} \leq \frac{\Delta m^2}{2m_\nu} \frac{\rho_m}{2m_\phi^2 \langle \phi \rangle^2} \tag{12}$$

i.e when the radion is light and satisfies the following inequality

$$m_\phi^2 \leq \frac{\rho_m}{\langle \phi \rangle^2} \frac{E_\nu}{m_\nu} \tag{13}$$

The most stringent bounds on m_ϕ can therefore be obtained by considering situations where the neutrino energy and the matter density are large. These criteria are ideally met by considering neutrino oscillations in supernovae where matter density $\rho_m \sim 10^{14} gm/cm^3$. The neutrino energies are in the range of $E_\nu = 50MeV$. Taking $m_\nu \leq 1$ eV we obtain the bound $m_\phi > 1$ GeV in order that the neutrino oscillations in supernovae be unaffected by the moduli exchange potential. In the supernovae the normal MSW resonance [4,5] due to standard weak interactions occurs in a cosmologicaly interesting mass range of $\Delta m^2 \sim 1$ eV. If the moduli mass is less than 1 GeV then the resonant conversions of electron neutrinos to other species will not occur as a (in eqn. 10) will be negative and there will be no resonance .

In supernovae neutrino oscillations are needed to explain the transfer of energy to the outer envelopes by the Bethe-Wilson mechanism [6], and also to explain the r-process nucleosynthesis [7]. If the moduli mass is smaller than a 1 GeV then these cannot take place within a cosmologically acceptable mass range of the neutrinos.

The neutrino oscillation induced by the Randall-Sundrum radion discussed in this report is a matter induced effect like the well known MSW mechanism. It vanishes if the neutrinos are degenrate but not massless. In

this respect it differs from the Damour-Polyakov (DP) dilaton [8] which occurs in the in the effective theory of gravity derived from strings. Like the RS radion it also couples to the trace of the energy momentum tensor. The neutrino oscillation induced by the DP dilaton does not vanish [9] if the neutrinos are completely degenerate but not massless. Because of this property the neutrino oscillation induced by the DP dilaton has been proposed as a possible solution to the solar neutrino problem. We would like to note that the radion coupling to visible brane matter given by eqn(2) has been derived by neglecting the back reaction of the bulk scalar (responsible for stabilizing the modulus) on the background metric. In this approximation the metric on the visible brane is given by $(\frac{\phi}{f})^2\eta_{\mu\nu}$. If the backreaction of the bulk scalar on the background metric is taken into account the resulting warp factor might depend on the radion field in a different way. This might change the radion coupling to visible brane matter. It would be interesting to examine the effects of such changes on the estimates presented in this report.

IV. Comparing the supernova bound on m_ϕ with bounds from B decays

In this section we shall present the bounds on light radion mass between $2m_\mu$ and $2m_\tau$ that could be derived from B decays. The radion coupling to SM fermions is similar to that of the Higgs coupling except for a scale factor of $\frac{v}{\langle\phi\rangle}$. Here v is the electroweak symmetry breaking scale. Therefore these bounds can be derived from similar bounds on m_h obtained from B decays by inserting the appropriate scale factor. In the following we shall assume that $\langle\phi\rangle = 1$ TeV.

Bounds on very light radion can be obtained both from inclusive and exclusive B deacys. However in this work we shall consider the bounds from inclusive B decays only since the hadronic uncertainties in their theoretical estimates are much less than those in exclusive decays. The most important inclusive B deacy limits on m_ϕ between $2m_\mu$ and $2m_\tau$ are as follows

1. Radion masses in the range $2m_\mu < m_\phi < 2m_\pi$ can be ruled out by the TASSO limits [10] on Br($\tilde{\phi} \rightarrow \mu^+\mu^- X$). In this mass range it is assumed that Br($\tilde{\phi} \rightarrow \mu^+\mu^-$) ≈1 which is valid to a very good approximation.

2. It may be possible to use the TASSO limit to rule out m_ϕ between 500 Mev and 1 Gev [11] if the $\tilde{\phi} \rightarrow \pi\pi$ enhancement does not severly suppress the $\tilde{\phi} \rightarrow \mu^+\mu^-$ branching ratio. However radion masses in the vicinity of the scalar resonance f_0(975 MeV) and other resonances in this region cannot be ruled out due to strong $\pi\pi$ final state interactions.

3. For 1 GeV$< m_\phi < 2m_\tau$ the CLEO limits [12] can be used to rule out existence of a radion so long as Br$(\tilde{\phi} \to \mu^+\mu^-) > 2.5 \times 10^{-3}$. The J/$\psi$ region is of course left out.

4. For $m_\phi > 2m_\tau, 2m_c$ the dominant deacys are $\tilde{\phi} \to \tau^+\tau^-$ and $c\bar{c}$ for which there are no reliable bounds.

We would like to note that the LEP and Tevatron bounds on radion mass presented in refs [13] and [14] do not exclude radions with mass between 100 MeV and 1 GeV. The LEP and Tevatron bounds on m_ϕ presented in these reports were derived based on $\tilde{\phi} \to b\bar{b}$ and $\tilde{\phi} \to \gamma\gamma$ decay modes of the radion respectively. However the branching ratios of these decay modes become negligible for radions with mass less than 1 GeV. This emhasizes the importance of excluding radions in the 100 MeV-1 GeV range by indirect methods like B decays and neutrino oscillation inside the supernova.

In conclusion in this report we have derived a lower bound on the radion mass so that radion exchange potential does not significantly affects the usual neutrino oscillation phenomenology inside the supernova. This bound is of the order of 1 GeV. Bounds on the radion mass of the same order of magnitude can also be derived from TASSO and CLEO limits on inclusive B decays.

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